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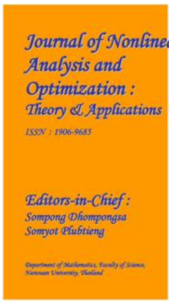

GENERALIZED THREE DIMENSIONAL FRACTIONAL FOURIER TRANSFORM AND SOME OF ITS APPLICATIONS

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Abstract :
The fractional Fourier transform (FrFT), a generalization of the Fourier transform, has been the focus of much research in recent years due to its applications in electronics and optics. The 1-D Fourier transform can be extended to the 1-D fractional Fourier transform (FrFT). Similarly, we can generalize a 3-D Fourier transform to a 3-D fractional Fourier transform (3DFrFT). Recently, some properties of FrFT have been developed by generalizing properties of regular Fourier transform (FrFT). In this review, we present applications of the generalized 3D fractional Fourier transform.

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1 Introduction

The Fractional Fourier Transform (FrFT) is a generalization of the Fourier Transform (FT). FrFT was first presented as a method for solving a certain class of ordinary and partial differential equations seen in quantum physics.[1] FrFT excels for analysing time-varying data, notably optics [2]. Iterative filtering, fractional convolution and correlation, beamforming, optional filters, convolution, filtering, wavelet transformations, and time-frequency representation are FrFT signal processing applications. FrFT enhances Fourier transform and frequency domain applications.[3] Recently, many researchers independently discussed FrFT. FrFT has been used in signal processing and optics, according to Ozaktas etal[4]. Fractional transformations were created by Alieva T[5] in the area of optical information processing. Recent progress in understanding the relationship between the fractional Fourier and linear canonical transformations has been presented by A. Bultheel et al [6].FrFT had been established as a signal processing tool by Djurovic etal [7]. Salazar F. [8] introduced the FrFT and its uses in an unique manner.[9-11] proposed a two-dimensional fractional Fourier transform to obtain generalized results for some elementary functions.Sinha A.[12] presented Three Dimensional Fractional Fourier-Mellin Transform and Applications.

2 Preliminaries

Definition 2.1 Fourier Transfrom : Fourier Transfrom of the function $g(t)$ is defined as

$$g(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(w) e^{jwt} dt$$

and invese is given by

$$g(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} g(w) e^{-jwt} dw$$

Definition 2.2 One Dimensional Fractional Fourier Transform : One dimensional Fractional Fourier transform with parameter α of $g(t)$ defined as

$$FrFT\{g(t)\} = G_\alpha(l) = \int_{-\infty}^{\infty} g(x)K_\alpha(t,l)dt$$

where the kernel

$$K_\alpha(t,l) = \sqrt{\frac{1-icot\alpha}{2\pi}}\ e^{\frac{i}{2sin\alpha}[(t^2+l^2)cos\alpha-2(tl)]}$$

Definition 2.3 Two Dimensional Fractional Fourier Transform : The two dimensional Fractional Fourier transform with parameter α of $g(t_1,t_2)$ defined as [10]

$$FrFT\{g(t_1,t_2)\} = G_\alpha(l_1,l_2) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t_1,t_2)K_\alpha(t_1,t_2,l_1,l_2)dt_1dt_2$$

where the kernel $K_\alpha(t_1,t_2,l_1,l_2) = \sqrt{\frac{1-icot\alpha}{2\pi}}\ e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+l_1^2+l_2^2)cos\alpha-2(t_1l_1+t_2l_2)]}$

3 Distributional Three Dimensional Fractional Fourier Transform

3.1 Conventional Three-Dimensional Fractional Fourier Transform

The two diemnsional Fractional Fourier transform with parameter α of $g(t_1,t_2,t_3)$ defined as

$$FrFT\{g(t_1,t_2,t_3)\} = G_\alpha(l_1,l_2,l_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(t_1,t_2,t_3)K_\alpha(t_1,t_2,t_3,l_1,l_2,l_3)dt_1dt_2dt_3$$

where the kernel

$$K_\alpha(t_1,t_2,t_3,l_1,l_2,l_3) = \sqrt{\frac{1-icot\alpha}{2\pi}}\ e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}$$

3.2 The Test Function Space E

An infinitely differentiable complex valued function ψ on R^n belong to $E(R^n)$ if for each compact set $A \subset S_{p,q,r}$, where

$S_{p,q,r} = \{t_1,t_2,t_3: t_1,t_2,t_3 \in R^n, |t_1| \le p, |t_2| \le q, |t_3| \le r,\quad p,q,r > 0\}, A \in R^n$

$\Upsilon_{E,a,b,c}(\psi) = Sup_{t_1,t_2,t_3}|D^{a,b,c}_{t_1,t_2,t_3}\psi(t_1,t_2,t_3)| < \infty$ Where, $a,b,c = 1,2,3,4,5\ldots$.

Thus $E(R^n)$ will denote the space of all $\psi \in E(R^n)$ with support contanined in $S_{p,q,r}$.

The space E is thus a Frechet space, since it is complete. Also if g is a member of E^*, the dual space of E, it is a fractional Fourier transformable.

3.3 Distributional Three Dimensional Fractional Fourier Transform

The three dimensional fractional fourier transform of $g(t_1,t_2,t_3) \in E(R^n)$ can be defined as $FRFT\{G(t_1,t_2,t_3)\} = G_\alpha(l_1,l_2,l_3) = \langle g(t_1,t_2,t_3), K_\alpha(t_1,t_2,t_3,l_1,l_2,l_3)\rangle$

where $K_\alpha(t_1,t_2,t_3,l_1,l_2,l_3) = \sqrt{\frac{1-icot\alpha}{2\pi}}\ e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}$

The right hand side of above has a meaning as the application of $g \in E^*$ to $K_\alpha(t_1,t_2,t_3,l_1,l_2,l_3) \in E$.

4 Examples on Generalized 3DFrFT

4.1 Prove that $[\mathbf{3DFrFT(1)}](\mathbf{l_1,l_2,l_3}) = \sqrt{\frac{\mathbf{i-icot\alpha}}{\mathbf{cot^3\alpha}}}\ \mathbf{e^{\frac{i3\pi}{4}}\ 2\pi\ e^{\frac{i}{2}(\frac{3+cos2\alpha}{sin2\alpha})(l_1^2+l_2^2+l_3^2)}}$

Proof :

$[3DFrFTg(t_1,t_2,t_3)](l_1,l_2,l_3) =$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\ e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}g(t_1,t_2,t_3)dt_1dt_2dt_3$$

$[3DFrFT(1)](l_1,l_2,l_3) =$

$$C_{1\alpha}\ e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(t_1^2+t_2^2+t_3^2)cot\alpha-i(t_1l_1+t_2l_2+t_3l_3)cosec\alpha}\ 1\ dt_1dt_2dt_3$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{icot\alpha(l_1^2+l_2^2+l_3^2)}{2}} \int_{-\infty}^{\infty} e^{(\frac{icot\alpha t_1^2}{2}-il_1t_1xcosec\alpha)}dt_1 \int_{-\infty}^{\infty} e^{(\frac{icot\alpha t_2^2}{2}-il_2t_2cosec\alpha)}dt_2 \int_{-\infty}^{\infty} e^{(\frac{icot\alpha t_3^2}{2}-il_3t_3cosec\alpha)}dt_3$$

where $C_{1\alpha} = \sqrt{\frac{(1-icot\alpha)}{2\pi}}$

We have

where $a = \frac{cot\alpha}{2}$, $b = -l_1cosec\alpha$

$$= C_{1\alpha}e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{[\frac{i(-l_1cosec\alpha)^2}{4(\frac{cot\alpha}{2})}]}\right]\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{[\frac{i(-l_2cosec\alpha)^2}{4(\frac{cot\alpha}{2})}]}\right]\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{[\frac{i(-l_3cosec\alpha)^2}{4(\frac{cot\alpha}{2})}]}\right]$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\right]^3 e^{\frac{i[(l_1^2+l_2^2+l_3^2)cosec^2\alpha]}{2cot\alpha}}$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\right]^3 e^{\frac{i}{2}l_1^2(\frac{cos\alpha}{sin\alpha}+\frac{1}{sin\alpha cos\alpha})}\; e^{\frac{i}{2}l_2^2(\frac{cos\alpha}{sin\alpha}+\frac{1}{sin\alpha cos\alpha})}\; e^{\frac{i}{2}l_3^2(\frac{cos\alpha}{sin\alpha}+\frac{1}{sin\alpha cos\alpha})}$$

$$= \sqrt{\frac{1-icot\alpha}{cot^3\alpha}}\; 2\pi\, e^{(\frac{3i\pi}{4})}e^{[\frac{i}{2}(\frac{cos2\alpha+3}{sin2\alpha})(l_1^2+l_2^2+l_3^2)]}$$

$$\therefore [3DFrFT(1)](l_1,l_2,l_3) = \sqrt{\frac{i-icot\alpha}{cot^3\alpha}}\; e^{\frac{i3\pi}{4}}\; 2\pi\; e^{\frac{i}{2}(\frac{3+cos2\alpha}{sin2\alpha})(l_1^2+l_2^2+l_3^2)}$$

4.2 Prove that :

$$[3DFrFT\;\delta(t_1-a,t_2-b,t_3-c)](l_1,l_2,l_3) = \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(a^2+l_1^2+b^2+l_2^2+c^2+l_3^2)cos\alpha-2(al_1+bl_2+cl_3)]}$$

Proof:

$$[3DFrFTg(t_1,t_2,t_3)](l_1,l_2,l_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}g(t_1,t_2,t_3)dt_1dt_2dt_3$$

$$[3DFrFT\;\delta(t_1-a,t_2-b,t_3-c)](l_1,l_2,l_3) = \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(t_1^2+t_2^2+t_3^2)cot\alpha-i(t_1l_1+t_2l_2+t_3l_3)cosec\alpha}\;\delta(t_1-a,t_2-b,t_3-c)dt_1dt_2dt_3$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\; e^{\frac{i}{2}(a^2+b^2+c^2)cot\alpha-i(al_1-bl_2-cl_3)cosec\alpha}$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(a^2+l_1^2+b^2+l_2^2+c^2+l_3^2)cos\alpha-2(al_1+bl_2+cl_3)]}$$

4.3 Prove that : $\mathbf{3DFrFT\; e^{i(at_1+bt_2+ct_3)}](l_1,l_2,l_3)} =$

$$\sqrt{\frac{1-icot\alpha}{cot\alpha}}\;\mathbf{2\pi\; tan\alpha\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2cot\alpha}[\frac{(a^2+b^2+c^2)-2cosec\alpha(al_1+bl_2+cl_3)}{cot\alpha}]+[\frac{(cos2\alpha+3)}{sin2\alpha}](l_1^2+l_2^2+l_3^2)}}$$

Proof:

$$[3DFrFTg(t_1,t_2,t_3)](l_1,l_2,l_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}g(t_1,t_2,t_3)dt_1dt_2dt_3$$

$$[3DFrFT\; e^{i(at_1+bt_2+ct_3)}](l_1,l_2,l_3) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]}\; e^{i(at_1+bt_2+ct_3)}\; dt_1dt_2dt_3$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha} \int_{-\infty}^{\infty} e^{(it_1^2cot\alpha+it_1(a-l_1cosec\alpha))}dt_1 \int_{-\infty}^{\infty} e^{(it_2^2cot\alpha+it_2(b-l_2cosec\alpha))}dt_2 \int_{-\infty}^{\infty} e^{(it_3^2cot\alpha+it_3(c-l_3cosec\alpha))}dt_3$$

Taking $a = \frac{cot\alpha}{2}, b = a - l_1 coes\alpha$ etc.

Therefore using equation (2)

$$[3DFrFT\; e^{i(at_1+bt_2+ct_3)}](l_1,l_2,l_3)$$

$$= \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha} \left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{\left[\frac{i(a-l_1cosec\alpha)^2}{4\left(\frac{cot\alpha}{2}\right)}\right]}\right]\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{\left[\frac{i(b-l_2cosec\alpha)^2}{4\left(\frac{cot\alpha}{2}\right)}\right]}\right]\left[\frac{e^{\frac{i\pi}{4}}\sqrt{\pi}}{\sqrt{\frac{cot\alpha}{2}}}\; e^{\left[\frac{i(c-l_3cosec\alpha)^2}{4\left(\frac{cot\alpha}{2}\right)}\right]}\right]$$

$$= \sqrt{\frac{1-icot\alpha}{cot\alpha}}\; 2\pi\, tan\alpha\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2}[l_1^2cot\alpha+\frac{(a-l_1cosec\alpha)^2}{cot\alpha}]}\; e^{\frac{i}{2}[l_2^2cot\alpha+\frac{(b-l_2cosec\alpha)^2}{cot\alpha}]}\; e^{\frac{i}{2}[l_3^2cot\alpha+\frac{(c-l_3cosec\alpha)^2}{cot\alpha}]}$$

$$= \sqrt{\frac{1-icot\alpha}{cot\alpha}}\; 2\pi\, tan\alpha\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2}[l_1^2cot\alpha+\frac{(a-l_1cosec\alpha)^2}{cot\alpha}]}\; e^{\frac{i}{2}[l_2^2cot\alpha+\frac{(b-l_2cosec\alpha)^2}{cot\alpha}]}\; e^{\frac{i}{2}[l_3^2cot\alpha+\frac{(c-l_3cosec\alpha)^2}{cot\alpha}]}$$

$$= \sqrt{\frac{1-icot\alpha}{cot\alpha}}\; 2\pi\, tan\alpha\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2cot\alpha}[\frac{(a^2+b^2+c^2)-2cosec\alpha(al_1+bl_2+cl_3)}{cot\alpha}]+[\frac{(cos2\alpha+3)}{sin2\alpha}](l_1^2+l_2^2+l_3^2)}$$

$$= \sqrt{\frac{1-icot\alpha}{cot\alpha}}\; 2\pi\, tan\alpha\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2cot\alpha}[\frac{(a^2+b^2+c^2)-2cosec\alpha(al_1+bl_2+cl_3)}{cot\alpha}]+[\frac{(cos2\alpha+3)}{sin2\alpha}](l_1^2+l_2^2+l_3^2)}$$

4.4 Prove that :

$$\mathbf{[3DFrFT\; e^{i(at_1^2+bt_2^2+ct_3^2)}](l_1,l_2,l_3) =}$$

$$\mathbf{2\pi \sqrt{\frac{1-icot\alpha}{(cot\alpha+2a)(cot\alpha+2b)(cot\alpha+2c)}}\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\; e^{icosec^2\alpha\left[\frac{l_1^2}{(2cot\alpha+4a)}+\frac{l_2^2}{(2cot\alpha+4b)}+\frac{l_3^2}{(2cot\alpha+4c)}\right]}}$$

Proof:

$$[3DFrFT g(t_1,t_2,t_3)](l_1,l_2,l_3) =$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2sin\alpha}[(t_1^2+t_2^2+t_3^2+l_1^2+l_2^2+l_3^2)cos\alpha-2(t_1l_1+t_2l_2+t_3l_3)]} g(t_1,t_2,t_3)dt_1dt_2dt_3$$

$$[3DFrFT\; e^{i(at_1^2+bt_2^2+ct_3^2)}](l_1,l_2,l_3) =$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}(t_1^2+t_2^2+t_3^2)cot\alpha-i(t_1l_1u+t_2l_2+t_3l_3)cosec\alpha}\; e^{i(at_1^2+bt_2^2+ct_3^2)}dt_1dt_2dt_3$$

$$=$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha} \int_{-\infty}^{\infty} e^{it_1^2[\frac{cot\alpha}{2}+a]-il_1cosec\alpha}dt_1 \int_{-\infty}^{\infty} e^{it_2^2[\frac{cot\alpha}{2}+b]-ivcosec\alpha}dt_2 \int_{-\infty}^{\infty} e^{it_3^2[\frac{cot\alpha}{2}+c]-iwcosec\alpha}dt_3$$

Taking $a = [\frac{cot\alpha}{2}+a], b = -l_1 cosec\alpha$ etc.

Therefore using equation (2)

$$[3DFrFT\; e^{i(at_1^2+bt_2^2+ct_3^2)}](l_1,l_2,l_3) =$$

$$\sqrt{\frac{1-icot\alpha}{2\pi}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha} \left[\frac{e^{\frac{i\pi}{4}}}{\sqrt{\frac{cot\alpha}{2}+a}}\sqrt{\pi}\; e^{\left[\frac{i(-l_1cosec\alpha)^2}{4(\frac{cot\alpha}{2}+a)}\right]}\right]\left[\frac{e^{\frac{i\pi}{4}}}{\sqrt{\frac{cot\alpha}{2}+b}}\sqrt{\pi}\; e^{\left[\frac{i(-l_2cosec\alpha)^2}{4(\frac{cot\alpha}{2}+b)}\right]}\right]\left[\frac{e^{\frac{i\pi}{4}}}{\sqrt{\frac{cot\alpha}{2}+c}}\sqrt{\pi}\; e^{\left[\frac{i(-l_3cosec\alpha)^2}{4(\frac{cot\alpha}{2}+c)}\right]}\right]$$

$$= \pi\sqrt{\frac{1-icot\alpha}{2}}\; \frac{e^{\frac{3i\pi}{4}}}{\left(\sqrt{\frac{cot\alpha}{2}+a}\right)\left(\sqrt{\frac{cot\alpha}{2}+b}\right)\left(\sqrt{\frac{cot\alpha}{2}+c}\right)}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\; e^{icosec^2\alpha\left[\frac{l_1^2}{(2cot\alpha+4a)}+\frac{l_2^2}{(2cot\alpha+4b)}+\frac{l_3^2}{(2cot\alpha+4c)}\right]}$$

$$= 2\pi\sqrt{\frac{1-icot\alpha}{(cot\alpha+2a)(cot\alpha+2b)(cot\alpha+2c)}}\; e^{\frac{3i\pi}{4}}\; e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)cot\alpha}\; e^{icosec^2\alpha\left[\frac{l_1^2}{(2cot\alpha+4a)}+\frac{l_2^2}{(2cot\alpha+4b)}+\frac{l_3^2}{(2cot\alpha+4c)}\right]}$$

3DFrFT of some functions

Sr. No.	$\mathbf{g(t_1, t_2, t_3)}$	$\mathbf{[3DFrFT\ g(t_1, t_2, t_3)](l_1, l_2, l_3)}$
1	1	$\sqrt{\frac{1-i\cot\alpha}{\cot^3\alpha}}\ e^{\frac{i3\pi}{4}}\ 2\pi\ e^{\frac{i}{2}\left(\frac{3+\cos 2\alpha}{\sin 2\alpha}\right)(l_1^2+l_2^2+l_3^2)}$
2	$\delta(t_1-a, t_2-b, t_3-c)$	$\sqrt{\frac{1-i\cot\alpha}{2\pi}}\ e^{\frac{i}{2\sin\alpha}[(a^2+l_1^2+b^2+l_2^2+c^2+l_3^2)\cos\alpha-2(al_1+bl_2+cl_3)]}$
3	$e^{i(at_1+bt_2+ct_3)}$	$\sqrt{\frac{1-i\cot\alpha}{\cot\alpha}}\ 2\pi\tan\alpha\ e^{\frac{3i\pi}{4}}\ e^{\frac{i}{2\cot\alpha}\left[\frac{(a^2+b^2+c^2)-2\operatorname{cosec}\alpha(au+bv+cw)}{\cot\alpha}\right]+\left[\frac{(\cos 2\alpha+3)}{\sin 2\alpha}\right](l_1^2+l_2^2+l_3^2)}$
4	$e^{i(at_1^2+bt_2^2+ct_3^2)}$	$2\pi\sqrt{\frac{1-i\cot\alpha}{(\cot\alpha+2a)(\cot\alpha+2b)(\cot\alpha+2c)}}\ e^{i\operatorname{cosec}^2\alpha\left[\frac{l_1^2}{(2\cot\alpha+4a)}+\frac{l_2^2}{(2\cot\alpha+4b)}+\frac{l_3^2}{(2\cot\alpha+4c)}\right]}$ $e^{\frac{3i\pi}{4}}\ e^{\frac{i}{2}(l_1^2+l_2^2+l_3^2)\cot\alpha}$

Conclusion

The current paper presents an extension of the 3D fractional Fourier transform.Some 3DFrFT applications are retained.

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